MATHEMATICAL STUDIES TZ1

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 15	16 – 29	30 – 40	41 – 54	55 – 67	68 – 79	80 – 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. As in May 2009, for the May 2010 examination session the IB has produced time zone variants of the Mathematical Studies papers. Grade boundaries for the different time zoned papers are set separately, and careful judgments are made that are based on criteria for performance level to account for differences in the papers.

Standard level project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 4	5 – 6	7 – 8	9 – 11	12 – 14	15 – 16	17 – 20

Range and suitability of work submitted

This session there was a diverse variety of topics. Statistical tasks still predominate but other areas such as modelling, measurement, financial mathematics, calculus, trigonometry and geometry were seen.

Many students included questionnaires and raw data, but a large number did not, or they organized and presented their data in ways which precluded cross-referencing of data and checking of mathematical processes.

Many candidates are now using technology to do the mathematics for them and often do not do any mathematics themselves. Any mathematical processes using technology only are considered simple. Some candidates used mathematical processes that were outside the syllabus. Generally this was not very successful as the mathematics seemed not well understood. Other candidates performed processes and then failed to comment on their results. This has the result of leaving the moderator to wonder whether or not the candidate really understands what they are doing.

When using the internet the candidate must remember to include the web address in their bibliography. More candidates are now including a bibliography.

The length of some projects was also a cause for concern. They varied from 1 or 2 pages to well over 50 pages. It is stated that the length of the project should not normally exceed 2000 words (excluding graphs, appendices and bibliography). There is no lower limit stated - but a project would have to contain several pages if it were to satisfy all the assessment criteria.

The comments made by the teachers on the 5/PJCS forms were very clear and helpful. Teachers are also encouraged to write on the projects and indicate where the mathematics has been checked for accuracy.

Candidate performance against the criteria

A. The statement of task was usually evident and most candidates described a plan that they would follow. It is important to actually follow the stated plan. If the plan is well documented, then the rest of the work tends to be better developed and follows a logical structure. Not all plans were well focused. Some projects did not have a title. Some candidates were clearly writing their plans after completing the project and used the past tense.

B. The majority of candidates collected their data and set it up in tables ready for the analysis. Some candidates had obviously collected data (via a questionnaire or otherwise) but omitted to include this data in their project. If the raw data is not present then the moderator cannot check the accuracy of the mathematical processes used. Data varied from 2 pieces of data to well over 100 pieces. The candidates must realise that having a lot of data does not always

mean that it has the quality needed to gain full marks in this section. If data is too simple and sample spaces are too small then it limits the mathematical analysis that the candidate can perform. When secondary information is used, candidates must clearly identify the source.

C. Many candidates only included simple mathematical processes in their projects. Many used technology only to perform sophisticated techniques without realizing that this is considered as simple mathematics. Some candidates introduced mathematical processes that were totally irrelevant. This can actually result in the candidate losing marks. Many candidates and their teachers are not clear on the chi-squared test. The entries in the contingency table must be frequencies and the expected frequencies must not be less than 1 and no more than 20% between 1 and 5. Otherwise the test is invalid.

D. Most candidates produced results that were consistent with their analysis. However, few produced detailed discussions. Often this was because the project was too simple to have much to say. The stronger candidates did a good job of presenting partial conclusions as they went along and then summarized these to give an overall conclusion at the end. It would be helpful if candidates assigned letters to all their tables and graphs and refer back to each and every one by number or letter in detail when discussing results.

E. Very few candidates are convincing in their understanding of the notion of validity. Their discussions generally centred on data collection. Less often was a student able to comment on the validity of the processes themselves.

F. Most of the projects were well laid out. Many candidates recorded their actions at each stage. It is important to ensure that the notation and terminology is correct. Many candidates lost marks this session due to errors in either notation or terminology.

G. The majority of the teachers appear to have awarded marks appropriately.

Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Give them examples of good projects so that they know what is expected of them.
- Make sure that they are aware of (and understand) the assessment criteria.
- Remind their students that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly.

- Tell them to include all raw data – but not all the completed questionnaires! A sample is sufficient as long as they gather all the data in organized tables
- Check that the mathematics used in the project is relevant
- Encourage the candidates to use more sophisticated mathematics
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Emphasise the importance of meeting deadlines
- Inform their students about sampling techniques
- Show their students how to use Equation editor or Math Type
- Check the calculations in each project
- Send the original work of the candidate to the moderator
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Write a comment to justify each achievement level awarded

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 26	27 – 36	37 – 48	49 – 60	61 – 72	73 – 90

General Comments

The paper appeared to be accessible, and of appropriate length. The comments on the G2 forms were mostly appreciative of the syllabus coverage, and of the level of difficulty.

The areas of the programme and examination which appeared difficult for candidates

The following tasks proved to be challenging for the candidates:

- Using the GDC to perform multi-step calculations
- Working with number sets
- Using the GDC to solve simultaneous equations
- Identifying a set described symbolically and shading it in a Venn diagram
- Completing logic tables for implications and equivalence
- Calculating probability for combined events and determining conditional probabilities
- Interpreting mapping diagrams and translating it to a function written algebraically
- Using the GDC to find the x-value of a parabola vertex
- Finding the amplitude, period, and frequency of sine and cosine functions from given graphs
- Using the GDC to solve equations, graphing unfamiliar functions
- Applying the sine and cosine rules
- Interpreting cumulative frequency diagrams
- Working with grouped data
- Using the p-value to make a decision about a stated hypothesis in a chi-squared test
- Working with multi-step currency conversions

Many students also had difficulty writing the equation of a straight line when the gradient and a point on the line are given, although it is a standard question. A great majority of the candidates had difficulty with drawing conclusions and writing clear, succinct, and well grounded justifications to support them.

The areas of the programme and examination in which candidates appeared well prepared

The majority of the candidates showed good time management skills and very few questions were not attempted. Good working was shown by the majority of the candidates so that follow through marks and method marks could be awarded when parts of questions were incorrect. Most scripts were neatly presented although still not all candidates are setting the work out carefully in the working box. It would be helpful for the examiner if candidates wrote the letter part of the question next to the working.

Volume and calculation of percentage error in a measurement were well understood as were the stem and leaf diagrams, rounding, scientific notation, Venn diagrams, currency conversions and use of a tree diagram to calculate probabilities of events. The degrees of

freedom for the chi-squared test were found correctly by most candidates and the null and the alternative hypotheses were mostly well stated. The sketch of the graph was mostly well done, although not all students used rulers.

Most candidates were able to demonstrate good knowledge of the learned mathematical concepts and their applications.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Exact volume and percentage error

This question was well answered by the majority of candidates. Candidates encountered difficulty in part (a) with using fractions finding the exact volume. Nearly all candidates could use the formula for volume and most could achieve at least 2 marks in this first part. A number of them lost the final mark because of a unit penalty for either not having any units in the answer or for using the incorrect units. Most candidates could find the percentage error correctly using the formula once they found the estimate for the volume. Very few candidates substituted the formula incorrectly, or had an incorrect denominator.

Question 2: Number sets

Very few candidates gained full marks in this question. A common error turned out to be that $\frac{22}{7}$ and 5×10^{-2} were not considered rational numbers. Also, 0 and $\sin(60^\circ)$ were often placed incorrectly. However, it was encouraging that very few candidates placed values in more than one region.

Question 3: Logic

The truth table was well done by the majority of candidates but significantly fewer could give the correct reason for whether the compound proposition was a tautology, so many lost 2 marks in this part of the question.

Question 4: Stem and leaf diagram

This question was very well answered by the majority of candidates. Some did not read the diagram key correctly and lost one mark for missing the decimal point. However, the mark scheme allowed them to gain the other marks as a follow through in the question. Some found incorrect quartiles but when showing the subtraction could gain one mark for knowing how to find the IQR.

Question 5: Volume and standard form

This question was well answered by many of the candidates. A significant number of candidates lost two marks in part (a) for not using the calculator correctly and omitting brackets in the denominator when entering the volume expression in their GDC. Also, those students who did not show the unrounded answer in the working box could not be awarded the last mark in part a). Follow through marks were awarded for parts (b) and (c) which most candidates gained.

Question 6: Venn diagram

This was probably the question that most candidates found the easiest. Nearly all candidates gained either 5 or 6 marks with the mark lost in shading the region on the Venn diagram.

Question 7: Trigonometry

Part (a) was generally well answered with many candidates gaining full marks. Some candidates went on to make incorrect assumptions about triangle BMC being right angled and used Pythagorus theorem incorrectly. Those who used either the Sine rule or the Cosine rule correctly were generally able to substitute correctly and gain at least two marks.

Question 8: Probability

Part (a) of this question was very well answered with many candidates gaining the maximum marks. Many candidates were less successful in part (b) and it seemed as if many of them either gained 3 marks or 0 marks. This shows that students who knew how to approach part b) were also able to correctly substitute in the formula they used and reach the correct answer. Very few of those students lost the last mark for wrong rounding.

Question 9: Cumulative frequency graph

This question was poorly answered by many of the candidates. A number of students could not identify the specified frequencies from the graph in part a). Most could not give the mid-interval value although surprisingly many of these candidates then went on and used the correct mid-interval value in the mean formula. A number did not understand the meaning of 'an estimate of the mean' and just wrote down a number read from the diagram.

Question 10: χ^2 test

The first two parts of this question were very well answered but a number of students found calculating the required expected value in part c) difficult. Very few knew how to use the given p-value in order to decide whether to reject or retain the null hypothesis. There were some candidates who did not attempt this question at all which might be indicating that this topic had not been discussed in some schools.

Question 11: Trigonometric function graphs

This question was a good discriminator between candidates with the best ones answering correctly and the weaker ones experiencing difficulties. There seemed to be confusion with this question. A few candidates found the period of the function in part a), and most had difficulties with finding the values of p and q in the function in parts b) and c). The use of the GDC to find the solution of the given equation in part d) was difficult for all but a few candidates.

Question 12: Currency Conversions

The vast majority of candidates answered at least part of this question with a significant number achieving full marks. A number did have a financial penalty applied for not giving their answers according to the specified accuracy level for the question. The most difficult part turned out to be (c) and a number of students didn't attempt it at all. There were very few candidates who used the incorrect conversion.

Question 13: Probability

Parts (a) and (b) were well answered with many candidates gaining 4 marks there. The conditional probability in part (c) proved to be more challenging. Nearly all candidates

attempted this question showing that time was not a factor in this paper. Many candidates gave their answers as incorrectly rounded decimals, which incurred an accuracy penalty and prevented them from gaining the maximum marks.

Question 14: Quadratic function

Most candidates attempted this question but very few of them completed it entirely. A number of students wrote incorrect equations in part (a), which shows that the mapping diagram was poorly understood and read. Part (c) proved to be difficult for many who didn't know how to find the x-coordinate of the vertex of the graph of the function. Some students gave the two coordinates instead of the x-coordinate only.

Question 15: Exponential function

Most candidates attempted this question and many gained 3 or 4 marks. All made an attempt at sketching the graph which demanded that students used their GDC. Many candidates failed to label their graphs and to give an indication of scale, and lost one mark in part (a). Some did not pay attention to the domain and sketched the graph in a different region. A significant number could also write down the coordinates of the y-intercept, although some wrote only $y = 2$ instead of giving the two coordinates. Almost all could draw the line $y = 5$ on the sketch but many could not find the answer for the number of solutions to the equation given in part c). Some candidates lost time in an attempt to draw this graph accurately on graph paper, which was not the intended task. Most candidates attempted this question, which clearly indicated that the time given for the paper was sufficient.

Recommendations and guidance for the teaching of future candidates

- Show working: All relevant working should be shown in each question with the question part indicated in the working box. Follow through marks can be then awarded where appropriate.
- When showing work, label the part of the question you are answering: Proper labeling is necessary as much to help your quick review at the end of the exam as for the examiners when they review and mark your work.
- Use GDC more effectively: Understand all the relevant functions and use of GDC. There is no need for explaining how the GDC was used, i.e. which keys were pressed, etc. Candidates need to be encouraged to use their GDC throughout the entire course.

Familiarity in using the calculators to graph unfamiliar functions and using it to solve equations is essential.

- Check answers carefully: Candidates should be reminded to check their answers to ensure they are reasonable in the context of the question.
- Pay attention to the required accuracy for the specific answers: Candidates should be reminded to give their answers to the accuracy required by a question, or to 3 significant figures otherwise. They must also know what penalties maybe applied if the accuracy is not achieved or the specified units not used.
- Know the command terms: Students should know all the command terms so that they know what action is required. They should also know the difference between "sketching a graph" and "drawing a graph," and invest the appropriate efforts in the given task.
- Learn to write succinct, clear, and well grounded justifications: It is important that students learn to communicate clearly. Teachers should model for students drawing conclusions and writing clear, succinct, and well grounded justifications to support them.
- Review past papers: Candidates should familiarize themselves with previous papers, their format, and key terms that are used.

Standard level paper two

Component Grade Boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 12	13 – 25	26 – 36	37 – 48	49 – 59	60 – 71	72 – 90

General Comments

Most candidates attempted all the questions, though there were a number of trivial attempts at question 3 or question 4. These trivial attempts were often seen as a whole centre event; the conclusion is that the candidates were not prepared for the entire syllabus and their marks suffered accordingly.

Lack of time did not seem to be an issue for the majority of candidates and the better ones were able to display their knowledge and skills, thereby achieving high marks. The examination was perhaps a little more challenging compared to other years, with the problems of a contextual nature proving most difficult for the candidates, but the paper, when considered together with paper 1, was deemed to be an appropriate test of the syllabus by the majority of teachers submitting G2 forms.

As ever, a number of candidates lost marks in the "show that" parts of the questions. It should be made clear to candidates that when they are required to reach a given answer written to a specified accuracy, they **must** first write down that that answer to a higher degree of accuracy than required by the question and **then** give the final answer as stated.

Marks continue to be lost through inaccurate reading of questions and lack of precision in answers, where the candidate is clearly competent with the associated mathematics; this is most notable where "coordinates" or "the *x* coordinate" or the "value of a function" might be asked for but where these instructions are ignored.

In the questions asking for angles it is becoming far less common to find candidates using their GDC in radians; this is an encouraging trend. It is also encouraging to see less need for the application of the financial and unit penalties.

The areas of the programme and examination which appeared difficult for candidates

- Scaling and labelling of accurate graphs.
- Application of the differential calculus.
- Trigonometry in 3 dimensional problems.
- Calculation of volume.
- Discrimination between the interest accrued on an investment and the value of that investment.
- Compound interest with compounding periods that are not annual.

The areas of the programme and examination in which candidates appeared well prepared

- Bivariate statistics.
- Differentiation by rule.
- Simple coordinate geometry.
- Simple arithmetic and geometric sequences and series.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Scatter Diagram

This question, by far, was the most accessible to the great majority of candidates. However, far too many candidates do not (1) use the scale as required by the question, (2) use a scale at all, (3) either draw or label axes, (4) use a ruler at all (5) use the provided graph paper. Accurate plotting of points can not be assessed unless graph paper has been used; the diagram is not a graph.

Many candidates did not seem aware that the regression line must pass through the mean point. Others, though they had obtained the equation of the regression line, did not use it to identify its *y* intercept.

Question 2: Coordinate Geometry

This question had many correct solutions, but a large number of candidates were unable to follow the logical flow of the question to the end and many gave up. It should be pointed out to future candidates that parts (e) and (f) could be attempted independently from the rest and that care must be taken not to abandon hope too early in the longer questions of paper 2.

Question 3: Calculus

This question was either very well done – by the majority – or very poor and incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, though it is recognised that the differential calculus is one of the more problematic topics for the candidature.

It was however disappointing to note the number of candidates who do not use the GDC to good effect; in part (a) for example, the zeros were not found accurately due to "trace" being used; this is not a suitable approach – there is a built-in zero finder which should be used. Much of the question was accessible via a GDC approach, a sketch was given that could have been verified on the GDC; this was lost on many.

Question 4: Trigonometry in 3 Dimensions, Volume

This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

Question 5: Finance and Arithmetic and Geometric Sequences and Series

For many, this question came as a welcome relief following the previous two questions. For those with a sound grasp of the topic, there were many very successful attempts.

Part A

A common error was to make all the comparisons using interest alone; though much credit was given for doing this, candidates should be aware of what is being asked for in the question.

The most common error was the use of 0.06 in the simple interest formula and 0.05 in the compound interest formula.

Many did not understand the notion of monthly compounding periods.

The financial penalty was frequently imposed, due to the lack of accuracy shown by some candidates.

Part B

A common weakness was seen in the "show that" parts of the question where, despite a lenient approach to method, many were unable to communicate their thoughts on paper.

For many, finding an expression for S_n in (c) was problematical.

The final part was challenging to the great majority, with a large number not attempting it at all; only the highly competent reached the correct answer.

Recommendations and guidance for the teaching of future candidates

- Ensure candidates can use the GDC efficiently – especially with regard to drawing graphs.
- Have the candidates begin each question on a new sheet of paper, show working, and space their answers in a logical fashion line by line; there is no benefit in trying to fit the entire examination on one sheet of paper.
- Draw graphs on graph paper.
- Teach time management – a mark a minute is the guide – ensuring that all questions are attempted and all parts of all questions are read.
- Cover the whole syllabus; it will all be examined.
- Practice with "show that" questions by having candidates communicate through their mathematics.
- Ensure candidates label and scale the axes, use a ruler and take a careful approach to their work whenever they draw a graph.
- Ensure candidates start each question on a new page and show all their working.
- Make the formula booklet a part of everyday teaching so that candidates become familiar with it.